Using the value of β to help determine γ from B decays

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Abstract

It has been pointed out by Gronau and Rosner that the angle γ of the unitarity triangle could be determined by combining future results on B_s and B_d decays to $K\pi$. Here we show that it is important to include in the analysis the information on the phase β which will be determined in the near future. Omitting this information could lead to an error as large as 8° in γ.

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A large number of experiments have been proposed to determine the phase $\gamma = Arg(V_{ub}^*)$ in the CKM matrix [1–8]. Before any of these experiments is completed it is likely that there will be a good measurement of $\sin 2\beta$. In many cases using the value of $\beta = Arg(V_{td}^*)$ derived from $\sin 2\beta$ can improve possible determinations of γ. We illustrate this for the case of a recent proposal by Gronau and Rosner [9] to determine γ by using U-spin symmetry (the exchange of s and d quarks) to relate the decays $B^0 \to K^+ \pi^-$ to $B_s \to K^- \pi^+$. Combining the rate of these decays with the rate for $B^+ \to K^0 \pi^+$ the value of γ could be obtained. We assume throughout the constraints of the CKM model.

The tree amplitude for B^0 (B_s) decay is proportional to $V_{ub}^* V_{ud}$ ($V_{ub}^* V_{us}$). The penguin amplitude is dominated by the virtual t quark and is proportional to $V_{tb}^* V_{ts}$ ($V_{tb}^* V_{td}$). Their approximation is to assume that the decay $B^+ \to K^0 \pi^+$ is purely penguin because only the penguin gives $b \to s \bar{d} d$. We then find for the decay amplitudes

$$A(B^+ \to K^0 \pi^+) = P, \tag{1a}$$

$$A(B^0 \to K^+ \pi^-) = T\, e^{i(\delta + \gamma)} + P, \tag{1b}$$

$$A(B_s \to K^- \pi^+) = \frac{1}{\tilde{\lambda}} T'\, e^{i(\delta' + \gamma)} - P' \left| \frac{V_{td}}{V_{ts}} \right| e^{-i\beta}; \tag{1c}$$

where $\tilde{\lambda} \equiv |V_{us}/V_{ud}| \simeq 0.226$. $|V_{td}/V_{ts}|$ is completely determined in terms of β, γ, and $\tilde{\lambda}$. The U-spin approximation is $P' = P$, $T' = T$, and $\delta' = \delta$.

In Ref. [9] unitarity is used to set

$$V_{tb}^* V_{ti} = - \left(V_{cb}^* V_{ci} + V_{ub}^* V_{ui} \right), \tag{2}$$

for $i = d, s$. Thus part of what we have called the penguin is now in the $V_{ub}^* V_{ui}$ term and combined with the tree; therefore, they get

$$A(B^+ \to K^0 \pi^+) = \bar{P}, \tag{3a}$$

$$A(B^0 \to K^+ \pi^-) = \bar{T}\, e^{i(\bar{\delta} + \gamma)} + \bar{P}, \tag{3b}$$

$$A(B_s \to K^- \pi^+) = \frac{1}{\tilde{\lambda}} \bar{T}'\, e^{i(\bar{\delta}' + \gamma)} - \tilde{\lambda} \bar{P}'; \tag{3c}$$

where $\bar{\delta}$ and $\bar{\delta}'$ are in general different from δ and δ' in Eqs. (1) and the last term follows since $V_{cd}/V_{cs} = -\tilde{\lambda}$. They thus obtain simple results independent of β.
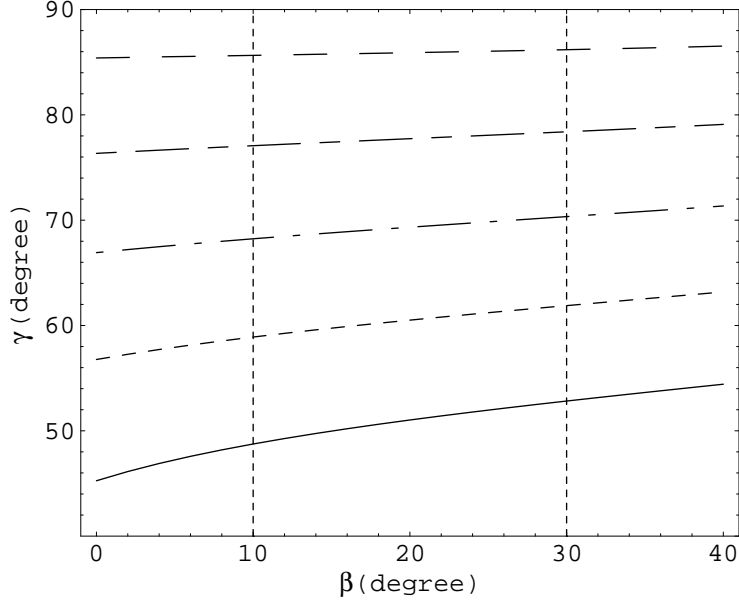
FIG. 1. *The dependence of γ on β for $R_d = 0.8$, $R_s = 0.78$ (solid line), $R_d = 0.85$, $R_s = 0.73$ (short dashed line), $R_d = 0.9$, $R_s = 0.68$ (dash-dot-dash line), $R_d = 0.95$, $R_s = 0.63$ (short-long dashed line), and $R_d = 1$, $R_s = 0.68$ (long dashed line), respectively. We assume $\cos\delta = 1$ and $r < 0$.*

However, terms of $\mathcal{O}(\tilde{\lambda}^2)$ and with dependence on both β and γ have been omitted from the first equation in (3). Since β will be known when this analysis can be used there is no purpose in eliminating β. We instead use Eqs. (1) to determine γ and the ratio $r \equiv P/T$ from the quantities R_d and R_s defined in Ref. [9] [1] for any value of β. Typical results are shown in Figs. 1 and 2 where we fix the sum of R_d and R_s and consider the limiting case $\delta = \delta' = 0$. The results of Ref. [9] are reproduced in the limit $\beta = 0$.

It is seen that for values of γ in the neighborhood of $50°$ and for $\beta = 30°$ ($\sin 2\beta = 0.87$) the values of γ is shifted from $\sim 45°$ to $\sim 53°$ from the $\beta = 0$ approximation. For values of γ in the neighborhood of $130°$ and for $\beta = 20°$ the shift is from $\sim 134°$ to $\sim 127°$. We assume $r < 0$ in accordance with the factorization assumption.

It is instructive to analyze the difference in the two approximations. The effective inter-

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[1] R_d is the ratio of the sum of B^0 and $\bar{B}^0$ decays to that of B^+ and B^- decays. R_s is the ratio of the sum of B_s and $\bar{B}_s$ decays to that of B^+ and B^- decays.
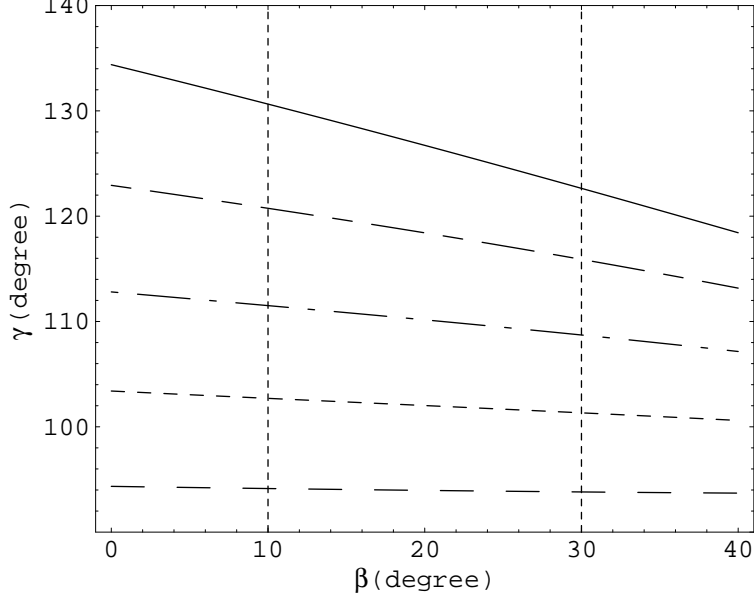
FIG. 2. *The dependence of γ on β for $R_d = 1.05$, $R_s = 0.53$ (long dashed line), $R_d = 1.1$, $R_s = 0.48$ (short dashed line), $R_d = 1.15$, $R_s = 0.43$ (dash-dot-dash line), $R_d = 1.2$, $R_s = 0.38$ (short-long dashed line), and $R_d = 1.25$, $R_s = 0.33$ (solid line), respectively. We assume $\cos \delta = 1$ and $r < 0$.*

action can be written as

$$\mathcal{H}_{eff} = V_{tb}^* V_{tq} \sum_{i=3}^{6} Q_i + V_{ub}^* V_{uq} \sum_{i=1}^{2} Q_i^{(u)} + V_{cb}^* V_{cq} \sum_{i=1}^{2} Q_i^{(c)}, \tag{4}$$

where $q = d$ or s and Q_i are the standard operators including the Wilson coefficients. We use the approximation that annihilation diagrams can be neglected so that $B^+ \rightarrow K^0 \pi^+$ is due to the penguin operators $Q_3 \sim Q_6$. Thus, as assumed in deriving Eqs. (1) the terms $P(P')$ are proportional to $V_{tb}^* V_{ts} (V_{tb}^* V_{td})$. In Ref. [9] unitarity is used to set

$$- V_{tb}^* V_{ts} = V_{cb}^* V_{cs} + V_{ub}^* V_{us} \tag{5}$$

and then the term proportional to $V_{ub}^* V_{us}$ is just omitted on the ground that it is smaller by a factor λ^2. In the limit that we neglect the strong phases we can include this term by replacing Eq. (3a) by

$$A(B^+ \rightarrow K^0 \, \pi^+) = \bar{P} \left[1 + \tilde{\lambda}^2 \frac{\sin \beta}{\sin(\beta + \gamma)} e^{i\gamma} \right]. \tag{6}$$

Formally our results reduce to theirs in the limit $\beta = 0$. It is the amplification of this factor $\tilde{\lambda}^2$ that is responsible for the difference.

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The equations of Ref. [9] for R_d and R_s become equations for $K R_d$ and $K R_s$, where

$$K = 1 + 2\tilde{\lambda}^2 \frac{\sin\beta \cos\gamma}{\sin(\beta+\gamma)} + \tilde{\lambda}^4 \left(\frac{\sin\beta}{\sin(\beta+\gamma)} \right)^2 . \tag{7}$$

The same factor K enters for R_d and R_s because both are defined as ratios to the B^+ decay. Then

$$K R_d = 1 + r^2 + 2r \cos\delta \cos\gamma,$$
$$K R_s = \tilde{\lambda}^2 + \left(\frac{r}{\tilde{\lambda}} \right)^2 - 2r \cos\delta \cos\gamma. \tag{8}$$

The amplification arises from the fact that $r \cos\gamma$ is proportional to $(K R_d - 1 - r^2)$. Thus, for example, with values of $R_d = 0.8$ and $R_s = 0.78$ (corresponding to $\gamma \sim 50°$) a change of K from 1 to 1.03 decreases $|r \cos\gamma|$ by about 10%. The value of r^2 is proportional to $\left[K(R_d + R_s) - (1 + \tilde{\lambda}^2) \right]$. For our example with $R_d + R_s = 1.58$ a change of K from 1 to 1.03 increases r by about 5%. Thus, a change of K from 1 to 1.03 can decrease $\cos\gamma$ by about 15%.

Unfortunately, the difficulty of using this method arises from the same sensitivity; small errors on R_d and R_s can cause a significant error on the determined γ. As an example, let the experimental errors be

$$\frac{\Delta R_s}{R_s} = 2 \frac{\Delta R_d}{R_d} \equiv 2\epsilon. \tag{9}$$

For the case shown in Fig. 1 with $\beta = 30°$ and $\gamma = 53°$, a value of $\epsilon = 6\%$ corresponds to an uncertainty of about 24% in $\cos\gamma$, yielding a value $\gamma = 53° \pm 10°$. For another case in Fig. 2 with $\beta = 18°$ and $\gamma = 128°$ and assuming instead $\Delta R_s/R_s = 4\Delta R_d/R_d \equiv 4\epsilon$, the same value of ϵ would correspond to an error of about 23% in $\cos\gamma$ and $\gamma = 128° \pm 10°$.

The accuracy of this method requires including the strong phase δ. In principle this can be determined by measuring the asymmetry between the rates for B^0 and $\bar{B}^0$, which is proportional to $\sin\gamma \sin\delta$. To a first approximation, the quantity that is determined in the method discussed here is $\cos\gamma \cos\delta$. Assuming δ is small probably only a limit on $\sin\gamma \sin\delta$ can be achieved. If $\cos^2\gamma < 1/2$ and $\sin\gamma \sin\delta < X$, then the uncertainty in δ leads to an

error of no more than $0.35\,X^2$ in $\cos\gamma$. It should be emphasized that this method depends upon the assumption that the sign of r is as given by factorization.

The approximation of neglecting contributions from Q_1 and Q_2 needs to be considered. The contribution of $Q_i^{(c)}$ can be included in $\bar{P}$ since in going from Eqs. (3a) to (3b) all that is required is that $\bar{P}$ corresponds to no change in isospin. As a result the only effect is a correction to the term proportional to $\tilde{\lambda}^2$ in Eq. (6). The contributions to Q_1 and Q_2 are long-distance effects due to rescattering which mixes processes of different topologies; calculations of these effects are very model dependent [10–12]. If we call $P_u(P_c)$ the amplitudes due to $Q_1^{(u)}+Q_2^{(u)}(Q_1^{(c)}+Q_2^{(c)})$ then the $\tilde{\lambda}^2$ terms in Eq. (6) must be multiplied by $1+(P_u-P_c)/\bar{P}$. Ciuchini *et. al.* [11], who call P_c the "charming penguin", suggest that $P_c/\bar{P}$ could be of order unity and Falk *et. al.* [12] suggest that $P_u/\bar{P}$ could be large. However, a recent analysis by Kamal [13] suggests that $(P_u - P_c)/\bar{P}$ is probably of order 0.1. As pointed out in these papers, it should be possible in the future to limit the values of P_u and P_c by detecting decays where they would make a major contribution.

In conclusion we emphasize that in determining γ from future experiments, optimum use should take into account the value of β which will be measured via $\sin 2\beta$ in the near future. In the examples we have discussed of B_d (B_s) decays to $K\pi$, the omission of the β dependence could lead to an error as large as $8°$ in special cases. In the longer run it would be valuable to determine the phase of the penguin amplitude and the phase 2β of the mixing independently so as to detect new physics contributions. Here we have limited the discussion to the standard CKM model.

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